A model for the source of the baryonic Tully–Fisher relationship and the Pioneer anomaly

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Abstract: It is shown that basic hypotheses regarding the nature of the source of the observed gravitational anomalies lead naturally to both the baryonic Tully–Fisher relationship and the Pioneer anomaly. A theory where the gravitational field of a mass induces a gravitational field contribution from the cosmos fits well with these hypotheses. In line with this, a theory is presented where particle–antiparticle pairs exist throughout the cosmos, with a lifetime in accordance with Heisenberg's uncertainty principle. These particle–antiparticle pairs are displaced in the presence of a gravitational field and contribute to the overall gravitational field of a given mass. The modeled contribution agrees with the baryonic Tully–Fisher relationship, the Pioneer anomaly, and the absence of any anomalous gravitational field within the inner solar system.

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Résumé : Nous montrons que les hypothèses de base sur la nature de la source des anomalies gravitationnelles observées mènent naturellement à la fois aux relations baryoniques de Tully-Fisher et à l'anomalie de Pioneer. Une théorie où le champ gravitationnel d'une masse induit une contribution au champ gravitationnel en provenance du cosmos agrée avec ces hypothèses. Suivant cette idée, nous présentons une théorie où les paires particule-antiparticule existent à travers le cosmos, avec un temps de vie en accord avec les relations de dispersion de Heisenberg. Ces paires de particule-antiparticule sont déplacées en présence d'un champ gravitationnel et contribuent au champ gravitationnel global d'une masse donnée. La contribution de modèle agrée avec la relation baryonique de Tully-Fisher, l'anomalie de Pioneer et l'absence de tout champ gravitationnel anomal dans la partie interne du système solaire.

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1. Introduction

The general theory of relativity and, in its weak field limit, Newton's law of universal gravitation have been found to provide results that are in excellent agreement with the motion of both natural and manmade objects within our solar system. In the weak field limit, the magnitude of the gravitational field, g_M, at a distance, r, from a mass, M, is given by

$$g_M = \frac{GM}{r^2} \tag{1}$$

where G is Newton's gravitational constant.

One possible exception to current gravitational theory has been found within our solar system: the Pioneer anomaly. Analysis of tracking data from the Pioneer 10 and 11 spacecrafts [1–3] has indicated the onset of anomalous acceleration directed towards the Sun at a distance between 10 and 15 ua (1 ua = 1.495 978 70 × 10^{11} m). From a distance of 15 ua to the limits of the data, approximately 50 ua from the Sun, this anomalous acceleration appears to be constant with a value of

$$a_{\text{PIONEER}} = (8.74 \pm 1.33) \times 10^{-10}\ \text{m s}^{-2} \tag{2}$$

Over this distance the gravitational field of the Sun, g_M, varies from 2.6 × 10^{-5} to 2.4 × 10^{-6} m s^{-2} so the effect is relatively small. Attempts to explain the anomalous acceleration as a result of forces generated by the spacecrafts themselves [3] have not as yet yielded values of large enough magnitude. Either an unknown gravitational source or a modification of gravitational theory, therefore, may be required to explain the Pioneer anomaly.

The Pioneer results within 15 ua of the Sun are somewhat inconclusive, for at these distances the solar radiation pressure on the Pioneer spacecraft is significant and the anomalous acceleration must be separated from the modeled acceleration due to the solar radiation pressure. For example, at a distance of 10 ua the additional acceleration of the spacecraft as a result of solar radiation pressure is approximately double that attributed to the anomalous acceleration and of opposite sign, as given by (2). On the other hand, considering that planetary ephemerides and the results from the Viking mission [2] and the Cassini mission [4] indicate that, at least within the inner solar system (i.e., <10–15 ua from the Sun), no anomalous acceleration seems to exist, the apparent onset of the Pioneer anomaly at 10–15 ua would appear to be a real effect.

It is when we move beyond our solar system that much larger anomalies are found between current gravitational theory and observation. Specifically, the orbiting stars within galaxies are found to be traveling at much greater velocities than is predicted by theory. For example, at the location of

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the Sun, approximately 8 kpc (1 pc = 3.085 678 × 10^{16} m) from the galactic centre, the Sun's orbital velocity is found to be 220 km s^{-1}. This can be compared with 160 km s^{-1}, the estimated value if only the observed galactic mass were contributing to the gravitational field. In terms of the gravitational fields this equates to

$$g_{AS} = 0.92 \times 10^{-10}\,\mathrm{m\,s^{-2}} \quad \text{for} \quad g = 1.96 \times 10^{-10}\,\mathrm{m\,s^{-2}} \tag{3}$$

where g_{AS} is the additional anomalous gravitational field at the location of the Sun, and g is the total gravitational field at that location.

There are two general characteristics of observed anomalous stellar rotation velocities for galaxies. First, it is found that stellar orbital velocities do not fall off with increasing distance as current gravitational theory predicts, but remarkably, as one moves further from the galactic center, the stellar rotational velocity curves flatten out, and the stellar velocities remain relatively constant. Second, it is found that there is a relationship between this constant orbital velocity found at large r and the total luminosity of the galaxy. This is called the Tully–Fisher [5] relationship. By taking into account the gas content of galaxies, it has been shown [6] that this relationship is fundamentally a relationship between the rotational velocity of the galaxy, v, and its total baryonic mass, M. This baryonic Tully–Fisher relationship is given by

$$M = (35\,\mathrm{s^4\,km^{-4}})M_S v^4 \tag{4a}$$

where M_S is the mass of the Sun and v is in units of km s^{-1}. Equation (4a) can also be expressed as

$$M = \left(6.97 \times 10^{19}\,\mathrm{kg\,s^4 m^{-4}}\right)v^4 \tag{4b}$$

where v is in units of m s^{-1}.

As an explanation for the larger-than-predicted stellar velocities, the assumption that the mass of galaxies resides primarily in the observable luminous matter, namely the stars, was brought into question [7]. A model of dark matter has thereby arisen, wherein it is currently believed that the major contribution to the mass of a galaxy, and thereby its gravitational field, are undetected nonbaryonic particles. These particles interact in such a manner that they only make their presence known by their gravitational effects on normal baryonic matter. The amount of dark matter required to reconcile observations with theory is astonishing, dominating baryonic matter in galaxies by up to one order of magnitude.

The dark matter theory does not require any modification to current gravitational theory. Dark matter particles, whatever they may be, contribute to the gravitational field as would any baryonic particle. The galactic rotational velocity curves are handled in the dark matter model by having the dark matter in a galaxy cluster with a density that varies with distance from the galactic nucleus as r^{-2}. The baryonic Tully–Fisher equation, (4b), is more difficult to explain as it implies a strong correlation between baryonic matter and dark matter.

As an alternative to the dark matter theory, several other theories have arisen. Modified Newtonian dynamics (MOND), proposed by Milgrom [8–10], postulates that the inertia of an object varies with acceleration such that, when placed in a gravitational field g_M, the acceleration of a mass, a_{MOND}, will be given by

$$a_{MOND} = \frac{g_M}{\mu(x)} \tag{5}$$

where $\mu(x)$ is a function of $x = |a_{MOND}|/a_0$ with a_0 being a constant to be determined. Different functions for $\mu(x)$ have been proposed. In the weak field limit, $g_M \ll a_0$, these functions are such that $\mu(x) \to x$, leading to

$$a_{MOND} = (a_0 g_M)^{1/2} \tag{6}$$

Substituting for g_M from (1) and v^2r^{-1} for a_{MOND} results in

$$M = \frac{v^4}{a_0 G} \tag{7}$$

in agreement with the baryonic Tully–Fisher relationship, as given by (4a). It has been surmised that the dependence of the inertia on acceleration (5) is due to a vacuum effect [11] that may also be responsible for the Pioneer anomaly [12].

Other alternatives to the dark matter theory include modified versions of the general theory of relativity such as modified gravitation (MOG) as proposed by Moffatt [13–15]. In summary, this theory finds that, far from the given mass, the gravitational force is stronger than predicted by Newton's gravitational theory, while at shorter distances, this effect is counteracted by a fifth force. MOG leads to a Yukawa-like modification of Newton's law of acceleration resulting in

$$a_{MOG} = g_M\left\{1 + \alpha\left[1 - (1 + \mu r)\mathrm{e}^{-\mu r}\right]\right\} \tag{8}$$

where the parameters α and μ are functions of the given mass, M. The theory has been used to explain the baryonic Tully–Fisher relationship [16] and the accompanying rotational curves of galaxies. Allowing the parameters to vary with distance from the given mass has also been shown to lead to results consistent with the Pioneer anomaly [17].

Another alternative to the dark matter model, as proposed by Blanchet [18–20], is the existence of a cosmic medium consisting of gravitational dipoles that become polarized in a gravitational field. This theory differs from both MOND and MOG in that no modification of current gravitational theory or Newtonian dynamics is required. The polarized dipole medium provides an additional contribution to the gravitational field surrounding a given mass. This model is in some sense similar to the dark matter model in that nonbaryonic particles, in this case gravitational dipoles, contribute to the gravitational field of a given mass. However, a major difference between this polarized cosmic medium model and the dark matter theory is that the field contribution of the gravitational dipoles, unlike dark matter particles, is dependent on the gravitational field due to the given mass.

The gravitational dipoles have been modeled [18] as consisting of a pair of particles, one of positive gravitational mass, and the other of negative gravitational mass, with both particles having positive inertial mass. The behavior of these gravitational dipoles in a gravitational field is treated as being analogous to electric dipoles in an electric field in that their dipole moments align with the gravitational field. An unspecified internal force between the pair of particles that constitute the dipole is required to bind the dipole together. This

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force is also required to balance the local gravitational field so that the dipole distribution about a given mass is time-independent. The internal force, and therefore the dipole moment, is modeled as being dependent on either the gravitational field [18, 19] or the polarization field [20]. The dependence is such that the model leads to the baryonic Tully–Fisher relationship.

There are two goals for this paper. The first goal is to show that basic hypotheses on the nature of the source of the gravitational anomalies lead naturally to both the baryonic Tully–Fisher relationship and the Pioneer anomaly. The second goal of this paper is to offer a new theory for the source of both the baryonic Tully–Fisher relationship and the Pioneer anomaly.

2. Theory

2.1 General behavior of the source of the gravitational anomalies

Without reference to the dark matter model or any of the alternative theories, consider the following two hypotheses with respect to the source of the gravitational anomalies. The first hypothesis is that whatever the source of the anomalous gravitational fields may be, its contribution, g_A, to the total gravitational field, g, is additive so that

$$g = g_M + g_A \tag{9}$$

The second hypothesis is that this additional field, g_A, is a function of the gravitational field of the given mass, that is, $g_A(g_M)$. Given that $g_M = g - g_A$, g_A can also be expressed as a function of the total field, that is, $g_A(g)$.

To determine the form of the function $g_A(g)$, consider the observational evidence. First, in the realm where the Pioneer anomaly arises, the gravitational field is relatively strong and the field due to the mass dominates the field due to the unknown source. Second, in the realm where the baryonic Tully–Fisher relationship holds, the gravitational field is relatively weak and the field due to the unknown source dominates the field due to the mass. This can be expressed as follows:

$$\text{for} \quad g \gg g_0 \qquad g_A \ll g_M \quad \text{and therefore} \quad g_A \ll g \tag{10a}$$

$$\text{and for} \quad g \ll g_0 \qquad g_A \gg g_M \quad \text{and therefore} \quad g_A \cong g \tag{10b}$$

where g_0 is a reference level to be determined. It is therefore hypothesized that the general behavior of the function $g_A(g)$ is as shown in Fig. 1, and further that $g_A(g)$ is a well-behaved monotonically increasing function that can be expanded as a power series

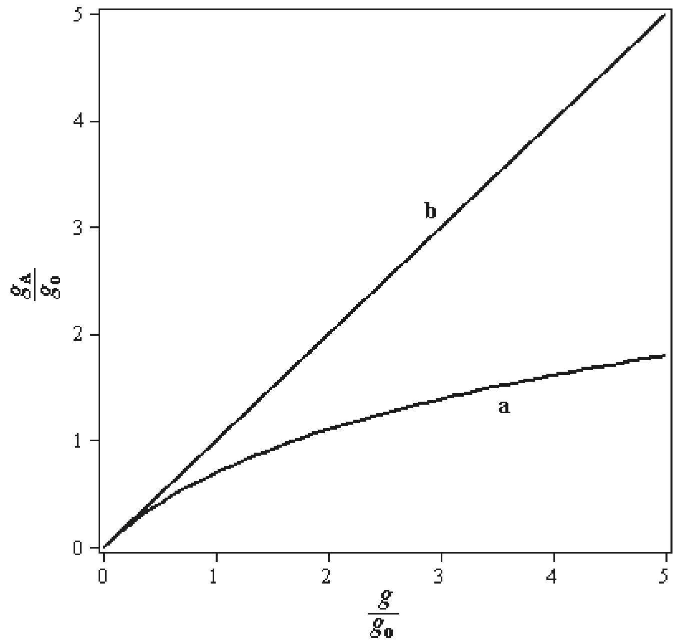
$$g_A/g_0 = \sum_i a_i (g/g_0)^i \tag{11}$$

Considering the general behavior given by (10a) and (10b), and as is shown in Fig. 1, the coefficient a_1 in (11) must equal one, and the coefficient a_2 must be negative. Setting a_2 equal to $-k$, (11) can then be expressed as

$$(g_A/g_0) = (g/g_0) - k(g/g_0)^2 + \mathcal{O}\left[(g/g_0)^3\right] \tag{12}$$

Substituting (12) into (9) then leads to

Fig. 1. (*a*) The general dependence that g_A is expected to have on the total field g; (*b*) the function $g_A = g$.



$$(g/g_0) = (g_M/g_0) + (g/g_0) - k(g/g_0)^2 + \mathcal{O}\left[(g/g_0)^3\right] \tag{13a}$$

which simplifies to

$$g_M/g_0 = k(g/g_0)^2 - \mathcal{O}\left[(g/g_0)^3\right] \tag{13b}$$

In the weak field limit where $g/g_0 \ll 1$, and substituting from (1) for g_M and $v^2 r^{-1}$ for g, (13b) becomes

$$M = \frac{k}{G g_0} v^4 \tag{14}$$

Equation (14) is of the same form as the baryonic Tully–Fisher relationship. The fact that this relationship falls out naturally from the two hypotheses is an indication that the hypotheses concerning the nature of g_A are correct. Equating the coefficients of (4b) and (14) results in

$$\frac{g_0}{k} = 2.15 \times 10^{-10}\ \mathrm{m\,s^{-2}} \tag{15}$$

Consider now, as examples, the following two functions that have the general behavior shown in Fig. 1 and as given by (10a) and (10b)

$$\begin{aligned} g_A/g_0 &= \ln\left[1 + (g/g_0)\right] \\ &= g/g_0 - 1/2(g/g_0)^2 + \mathcal{O}\left[(g/g_0)^3\right] \end{aligned} \tag{16a}$$

and

$$\begin{aligned} g_A/g_0 &= (1 - \mathrm{e}^{-g/g_0}) \\ &= g/g_0 - (1/2!)(g/g_0)^2 + \mathcal{O}\left[(g/g_0)^3\right] \end{aligned} \tag{16b}$$

For both of these functions, the value of k is equal to 1/2, and therefore, from (15), for both of these functions, $g_0 = 1.08 \times 10^{-10}$ m s^{-2}. Using this value for g_0, these two functions are shown in Fig. 2 along with the value of g_{AS} as given by (3). Both of these functions are seen to be in good

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Fig. 2. (*a*) $g_A = g_0 \ln[1 + (g/g_0)]$; (*b*) $g_A = g_0 (1 - e^{-g/g_0})$; (*c*) $g_A = g$; (*d*) The value of g_{AS} as given by (3).

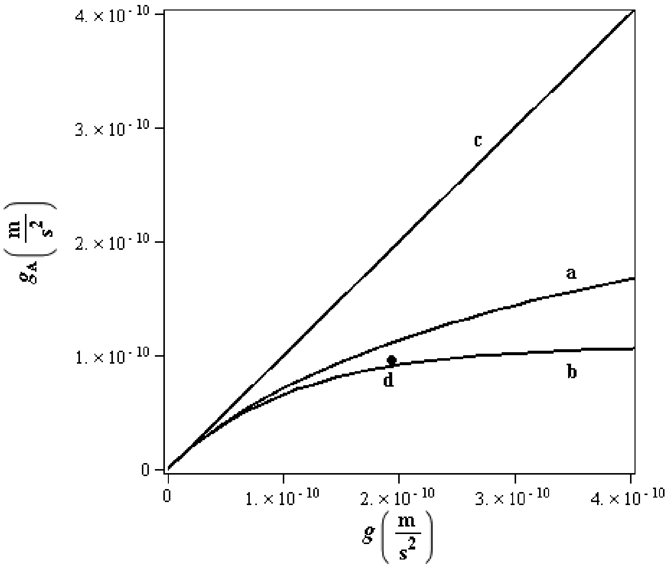
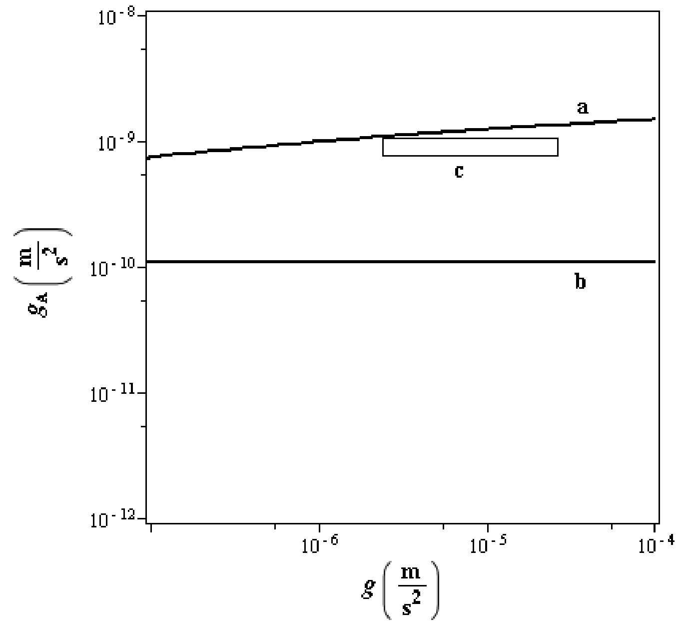


Fig. 3. (*a*) $g_A = g_0 \ln[1 + (g/g_0)]$; (*b*) $g_A = g_0 (1 - e^{-g/g_0})$, for the relatively strong fields in the vicinity of our Sun; (*c*) the range of the Pioneer anomaly, as given by (2).



agreement with the value of g_{AS}. Equations (16a) and (16b) are just examples of functions that have the general behavior of Fig. 1. In general, any function that has the behavior shown in Fig. 1 and that can be expressed as a power series as per (12) will result in agreement with the baryonic Tully–Fisher relationship.

Differences between functions, $g_A(g)$, will become evident in the case of stronger gravitational fields. In the realm of the Pioneer anomaly the gravitational fields are on the order of 10^4–10^6 times stronger than the fields where the baryonic Tully–Fisher relationship holds. Figure 3 shows the two example functions given by (16a) and (16b), over this range of g, along with the range of $a_{PIONEER}$, as given by (2). As can be seen, both functions lead to nearly constant values for g_A of approximately 1×10^{-9} and 1×10^{-10} m s^{-2}, respectively, over this range of g. These values are of the same order of magnitude as those found in the Pioneer anomaly, with (16a) being an especially good fit to the Pioneer values. It is important to stress again that the functions given by (16a) and (16b) are just examples. In general, for any monotonically increasing function that can be expanded as a power series, as per (12), and that has its second coefficient calibrated to the baryonic Tully–Fisher relationship, the functions' values, when extrapolated to the gravitational field in the vicinity of the Sun, will result in an approximately constant value for g_A that is of the same order of magnitude as the Pioneer anomaly. Ockham's razor would seem to indicate that the source of the Pioneer anomaly and the baryonic Tully–Fisher relationship is one and the same.

2.2 A model for the source of the gravitational anomalies

Based on their agreement with the baryonic Tully–Fisher relationship and the Pioneer anomaly, it is proposed that the two starting hypotheses regarding the nature of the source of the gravitational anomalies are valid. Specifically that the anomalous source's gravitational field adds to the gravitational field surrounding a given mass, g_M, and in addition that its gravitational field is a function of g_M. The dark matter model is in agreement with the first hypothesis, but given that, in the dark matter theory, the dark matter dominates baryonic matter, it would not be expected that the gravitational field due to the dark matter, g_A, would be dependent on the gravitational field due to the baryonic mass, g_M. Hence, the dark matter theory does not fit naturally with the two hypotheses. Although both the MOND and MOG theories may be able to be recast in the form of (9), that is, where the total gravitational field can be expressed as the Newtonian field plus an additional term dependent on the Newtonian field, this is not a natural fit for these two alternatives. On the other hand, any theory similar to Blanchet's, where the gravitational field of a given mass induces a contribution to the total gravitational field from the cosmic medium, will fit very well with both hypotheses. Such a theory will lead naturally to both a baryonic Tully–Fisher relationship for weak fields and a Pioneer-like anomaly for stronger fields.

In Blanchet's model, the cosmic medium is taken to consist of entities that have a gravitational dipole. These dipoles will align in the gravitational field of a given mass, and thereby contribute to the overall gravitational field. The gravitational dipoles are modeled to consist of a pair of particles, one of positive gravitational mass and the other having a negative gravitational mass. This model requires the existence of some unspecified internal force with specific characteristics so as to keep the dipoles bound and, in addition, to keep the gravitational dipole distribution stable with respect to the given mass. An alternative to this gravitational dipole model will now be presented.

First, consider the following speculation on the nature of the vacuum. If we take the two key features of a region of space that is considered to be a vacuum to be that it has no net energy and no net charge, then there are at least two ways that this can be achieved. The simplest possibility is that there exist no particles within this region. A second possibil-

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ity, however, is that particle–antiparticle pairs exist with total energy zero. This would correspond to the binding energy being equal in magnitude to the pair's rest mass energy plus any kinetic energy it may have. A region of space containing such entities would have a net charge of zero as well as a net energy of zero and thus would have the properties attributed to a vacuum.

Given the existence of such particle–antiparticle pairs in the vacuum, it is further hypothesized that the total energy of a given particle–antiparticle pair will not be exactly zero, but will have a value in accordance with Heisenberg's uncertainty principle

$$|E| \cong \frac{\hbar}{2\tau} \tag{17}$$

where τ is the lifetime of these entities. Here $|E|$ refers to the magnitude of the net energy of the given entity, as the net energy of the pair can be positive or negative. The overall average energy of the vacuum will still be zero, but with this model the vacuum would be populated by particle–antiparticle pairs, each with total energy either above or below zero.

Consider now the effect that a mass, with its accompanying gravitational field, will have on the vacuum. During a net positive energy pair's lifetime, the pair will be gravitationally attracted to the given mass. It is also hypothesized that a net negative energy pair will be repelled by the given mass. Although, with this model, there are no gravitational or energy dipoles per se in the vacuum, the displacement of a positive energy entity towards a given mass or a negative energy entity away from the mass is equivalent to an energy dipole aligning with the gravitational field. Given this behavior of the particle–antiparticle pairs, the contribution of the vacuum to the gravitational field of a mass is analogous to the contribution of a dielectric to the electric field of a charge.

An equivalent energy dipole moment, $\boldsymbol{p}_E$ (units of J m), can be defined for each particle–antiparticle pair. This vector will equal E, the net energy (positive or negative), of the given particle–antiparticle pair, multiplied by the time-averaged displacement, $\Delta\boldsymbol{r}$, of the pair during its temporal existence towards, if its net energy is positive, or away from, if its net energy is negative, the given mass

$$\boldsymbol{p}_E = E\Delta\boldsymbol{r} \tag{18}$$

The vector $\boldsymbol{p}_E$ will point towards the given mass for both a net positive energy pair and for a net negative energy pair. The energy dipole moment density of the vacuum, $\boldsymbol{P}_E$ (units of J m^{-2}), will then be given by

$$\boldsymbol{P}_E = N\tau\boldsymbol{p}_E \tag{19}$$

where N is equal to the rate per unit volume at which particle–antiparticle pairs, of either positive or negative energy, come into existence. As with the equivalent energy dipole moment, $\boldsymbol{p}_E$, the energy dipole moment density, $\boldsymbol{P}_E$, will point towards the given mass. From (17) and (18) the magnitude of $\boldsymbol{P}_E$ can be expressed as

$$\boldsymbol{P}_E = \frac{\hbar}{2}N\Delta\boldsymbol{r} \tag{20}$$

The energy density of the particle–antiparticle pairs, ρ_E (units of J m^{-3}), in the gravitational field of a given mass can then be determined from

$$\rho_E = -\nabla\cdot\boldsymbol{P}_E \tag{21}$$

The energy density given by (21) is a real energy density. When ρ_E is positive, this corresponds to there being more positive energy pairs than negative energy pairs in the region, while when ρ_E is negative this corresponds to there being more negative energy pairs in the region than positive. In the region surrounding a given mass, the energy density of the particle–antiparticle pairs will be positive. This positive energy density will contribute to the gravitational field of the mass that induced it as would any other source of energy. Given the temporal nature of the particle–antiparticle pairs, the energy density distribution, as given by (21), will be time independent.

Given (21), the gravitational field at a distance r from a given mass, due to the displacement of the particle–antiparticle pairs, will be given by

$$\boldsymbol{g}_{\mathrm{A}} = -\frac{G}{c^2r^2}\int \rho_E \mathrm{d}V\hat{\boldsymbol{r}} \tag{22a}$$

$$\boldsymbol{g}_{\mathrm{A}} = -\frac{G}{c^2r^2}\int (-\nabla\cdot\boldsymbol{P}_E)\mathrm{d}V\hat{\boldsymbol{r}} \tag{22b}$$

$$\boldsymbol{g}_{\mathrm{A}} = \frac{G}{c^2r^2}\int \boldsymbol{P}_E\cdot\mathrm{d}\boldsymbol{S}\hat{\boldsymbol{r}} \tag{22c}$$

where V is a spherical volume of radius r and S is the bounding surface. For an isotropic $\boldsymbol{P}_E$, (22c) becomes

$$\boldsymbol{g}_{\mathrm{A}} = \frac{4\pi G}{c^2}\boldsymbol{P}_E \tag{23}$$

The resulting gravitational field, $\boldsymbol{g}_{\mathrm{A}}$, due to the particle–antiparticle pairs, is therefore directly proportional to $\boldsymbol{P}_E$ and points in the direction of $\boldsymbol{P}_E$: towards the given mass.

In Sect. 2.1, it was shown that using (16a) as a model for g_{A} results in good agreement with the value of the Pioneer anomaly as given by (2). Using (16a) as a model for g_{A}, the magnitude of the energy dipole moment density will, by (23), be given by

$$P_{\mathrm{E}} = \frac{c^2}{4\pi G}g_0\ln\left[1+(g/g_0)\right] \tag{24}$$

If N is constant then, from (20) and

$$\Delta r = \frac{c^2}{2\pi\hbar GN}g_0\ln[1+(g/g_0)] \tag{25}$$

it would be expected that if the gravitational force was the only force acting on a given particle–antiparticle pair the pair's displacement towards the given mass would be given by

$$\Delta r = (1/2)g\overline{t^2} = (1/6)g\tau^2 \tag{26}$$

where Δr is directly proportional to the gravitational field, g. In the weak field limit, $g \ll g_0$, (25) simplifies to

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$$\Delta r = \frac{c^2}{2\pi\hbar GN} g \quad (27)$$

which does agree with the expected dependence on g. However, it is surmised that, as the gravitational field increases and the displacement of the particle–antiparticle pairs increases, interactions between neighboring particle–antiparticle entities come into play. These interactions lead to the displacement, Δr, being reduced from the value given by (27) to the value given by (25).

Equation (24) and the resulting relationship between g_A and g, as given by (16a), lead to good agreement with the value of the Pioneer anomaly as given by (2), as was shown in Fig. 3. However, the model does not agree with the onset of the anomaly at 10–15 ua, and the observations that indicate that no anomalous gravitational field exists within the inner solar system. Therefore, (16a) and (24) must only apply for gravitational fields below a certain magnitude.

It is surmised that, N, the rate at which the particle–antiparticle entities come into existence, is dependent on the value of the gravitational field. A gravitational field must in some manner hinder the coming into existence of the particle–antiparticle pairs in the model. As will be shown, the following function for N leads to excellent agreement with the observations

$$N = N_0 e^{-(g/g_1)^2} \quad (28)$$

where N_0 is the rate at which the particle–antiparticle entities come into existence when $g \to 0$, and g_1 is set equal to 4×10^{-5} m s^{-2}, corresponding to the gravitational field of the Sun at 12 ua. The corrected expression for the displacement, Δr, of these entities in a gravitational field is then

$$\Delta r = \frac{c^2}{2\pi\hbar GN_0} g_0 \ln[1 + (g/g_0)] \quad (29)$$

The expression for the energy dipole moment density, P_E, will then, by (20), be given by

$$P_E = \frac{c^2}{4\pi G} e^{-(g/g_1)^2} g_0 \ln[1 + (g/g_0)] \quad (30)$$

and the anomalous gravitational field, g_A, surrounding a given mass will, by (23), be given by

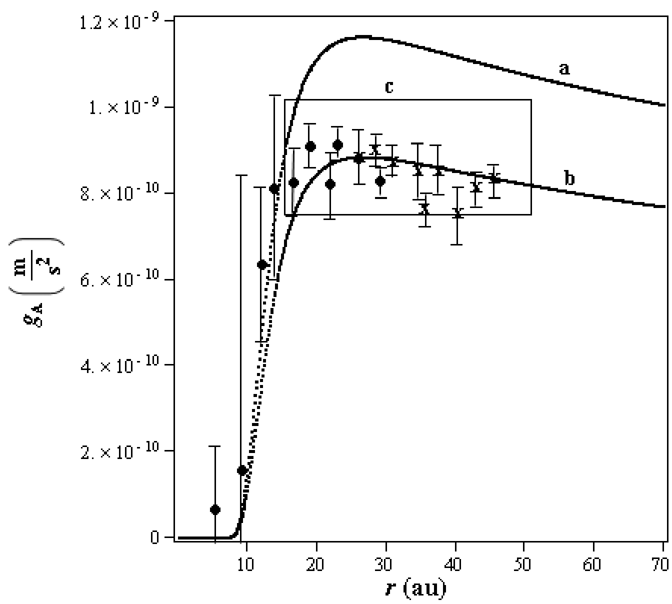
$$g_A = g_0 e^{-(g/g_1)^2} \ln[1 + (g/g_0)] \quad (31)$$

For the case of the gravitational field in the vicinity of the Sun, where $g \cong g_M = GM_S r^{-2}$, (31) can be expressed as

$$g_A = g_0 e^{-(GM_S/r^2 g_1)^2} \ln[1 + (GM_S/r^2 g_0)] \quad (32)$$

The dependence of g_A on r, as given by (32), for distances up to 70 ua from the Sun is shown in Fig. 4 along with the Pioneer results, including values within 15 ua. The general behavior of (32) matches observations within our solar system. The anomalous gravitational field is relatively constant between 15 and 70 ua with a value of approximately 1×10^{-9} m s^{-2}. This value then quickly drops to zero at distances between 15 and 10 ua. Also shown in Fig. 4 is the dependence of g_A on r for a value of g_0 equal to 8.0×10^{-11} m s^{-2}, corresponding to the coefficient of the baryonic Tully–Fisher relation in (4a) being increased from 35 to 47 s^4 km^{-4}. This particular value for g_0 leads to excellent agreement with the Pioneer results.

Fig. 4. (*a*) g_A versus r as per (32) with $g_0 = 1.08 \times 10^{-10}$ m s^{-2} and $g_1 = 4 \times 10^{-5}$ m s^{-2}; (*b*) as per (*a*), but with $g_0 = 8.0 \times 10^{-11}$ m s^{-2}; (*c*) The range of the Pioneer anomaly as given by (2); ● represent Pioneer 11 anomolous accelerations; x represent Pioneer 10 anomolous accelerations.



To summarize, (28) and (29) model the behavior of the hypothesized particle–antiparticle pairs in a gravitational field. Equation (28) gives the rate at which these entities come into existence and (29) gives their average displacement during their lifetime. These relationships lead to the energy dipole moment density given by (30) and the resulting anomalous gravitational field given by (31) and (32). This relationship between g_A and g as given by (31), in the case where $g_1 \gg g_0$, can be separated into three regions.

$g \ll g_0$.
: For these weak fields, (31) can be expanded as $g_A = g - (1/2)\, g_0 (g/g_0)^2 + \mathcal{O}[(g/g_0)^3]$. This, as was shown in Sect. 2.1, leads to the baryonic Tully–Fisher relationship; $M = (1/2Gg_0)v^4$. The coefficient of the baryonic Tully–Fisher relationship of (4b) then leads to a value for g_0 of 1.08×10^{-10} m s^{-2}.

$g_0 < g \ll g_1$.
: For these moderate fields, (31) is approximated by $g_A = g_0 \ln[1 + (g/g_0)]$. This region includes the gravitational field of the Sun at distances from 15 to 50 ua. The value of g_A over this range is found to be approximately constant with a value of $g_A = 1 \times 10^{-9}$ m s^{-2}.

$g > g_1$.
: For these stronger fields, as the value of g increases, (31) results in $g_A \to 0$. The onset of the Pioneer anomaly indicates that g_1 is approximately 4×10^{-5} m s^{-2}.

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4. Conclusion

The hypotheses that the source of the gravitational anomalies contributes a gravitational field, g_A; that adds to the gravitational field, g_M, of a given mass; is a function of g_M; and has the general behavior shown in Fig. 1, was shown to lead to both the baryonic Tully–Fisher relationship and a Pioneer-like anomaly. A theory where the gravitational field of a mass induces a gravitational field contribution from the cosmos fits well with these hypotheses. Such a model does not require any modification of Newtonian dynamics or of current gravitational theory.

The speculation on the existence of particle–antiparticle pairs throughout the cosmos with a lifetime given by Heisenberg's uncertainty principle is of course no more than speculation. Whether these pairs are connected to the virtual pairs of quantum field theory remains to be seen. Further work is required to determine the nature of these particle–antiparticle pairs. The behavior of these particle–antiparticle pairs in a gravitational field, as modeled by (28) and (29), leads to their contribution to the gravitational field of a given mass as given by (31). This contribution agrees with the baryonic Tully–Fisher relationship, the Pioneer anomaly, and the lack of any anomalous gravitational field within our inner solar system.

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